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Development of the method of Optimal Control by Technological process of Crushing

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Abstract— The problem of optimal control is qualified by process of crushing of initial materials, as a problem of consecutive decision-making which belongs to the class of statistical problems in research work. Process of search of optimum control was approximated Markov by casual process with discrete conditions and continuous time. The method of optimum control by process of crushing with the minimum square-law function of losses on each step of management is developed.

Index Terms— crushing, crushing sorting manufacture, casual process, Markov process, optimum control, prize functions, large a crushing product

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1 Introduction

In modern conditions for the purpose of increase of efficiency of processes of crushing and sorting there is a necessity of creation of the automated crush-sorting manufacture (ACSM), representing difficult multilevel system [1,2]. The difficult automated systems include a considerable quantity of the diverse elements united by means of branched out mutually intertwining communications for achievement of some ultimate goal. The organization of process of processing of the information circulating in similar systems, development on their basis of rational commands of management in interests of achievement of an object in view, a choice of the best mode of functioning of all elements and system as a whole is carried out by means of modern computers with the developed software. For the purpose of creation effective ACSM it is necessary to develop new mathematical methods for the decision of a problem of optimal control of process of crushing for the bottom level of technological process at stochastic character large the initial material, providing demanded value large a crushing product at the minimum losses of process Managements. Optimal control at automation of crushing manufacture should be directed on maintenance of demanded productivity and demanded fractional structure of a product of crushing [3,4].

Key parameters of management of crushing units are the size of a target crack and frequency of rolling a mobile cheek. Crushers or frequency of rotation of a splitting cone for cone crushers. With the size of a target crack and values of frequency, angular speed and eccentric mobile parts of crushing units it is directly connected target large scale a product of crushing and parameters of its law of distribution.

Therefore optimum control in these parameters should be carried to the main influences influencing quality of a product of crushing, demands carrying out wide theoretical and experimental researches for the purpose of reception of laws of management and ways of their realization in the automated system of crushing. Frequency rolling staff a crusher and angular speed of rotation - cone crushers can be regulated change Pressure of a food of an electric drive its simple inclusion or reenergizing, and, hence, to regulate large product crushing and, finally, productivity of a crusher (see [4]). Nevertheless, for automation of process of crushing it is desirable to have possibility of its more perfect regulation. However, on to the existing data, a scope of adjustable drives in crushers it is limited to experimental units thus. There is no systematized data about distribution laws Frequencies, angular speeds and eccentric the corresponding Units of crushers. It limits carrying out possibility Theoretical researches by definition of laws of regulation for Frequencies, angular speeds and eccentric mobile units the crushers providing, along with the size of a target crack of a crusher, receptions demanded large a crushing product. For this reason in the given work the basic attention is given research management of fractional structure of a product of crushing by regulation of the size of a target crack cheeks. And cone crushers at constant values of frequencies, angular speeds and clown mobile units in this connection there is a question on a finding of the best in this or that sense or as speak, optimal control of crushing process. An ultimate goal of management of crushing process, at the accepted assumptions, is reception of demanded value large scale a product of crushing which, as shown above, is the stochastic characteristic of a product of crushing. It means that in

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managerial process the automated system will have the information only about some average value large crushing product, and to suffer damage, owing to a deviation of its current value from demanded value. At practical realization of the automated system of an expense for management of process can force to search compromise parity between cost of management and damage owing to a deviation from demanded value large crushing product. From this point of view the management optimality is understood in sense of achievement with some accuracy of demanded value large crushing product at the minimum value of a total damage of management.

2 Mathematical Statement of the Problem

For the decision of a problem of optimal control, in the conditions considered above, it is possible to use results of the stochastic theory of management (regulation) with adaptation. Assumptions are put in a basis of this theory about process of search of optimal control as casual process without aftereffects (Markov) [5]. Statement of a problem for Markov processes of the decision with final number of steps can be formulated so. Let *S* designates space of conditions of consecutive *n*-step-by-step process so on any step the process condition can be described point $z \in S$, and it is possible to choose one alternative from the set class and at known function of prize r and transitive function *f*. If on some step process is in condition $x \in S$ and alternative $a \in A$, that this case is chosen it is possible to receive average prize $r(x, a)$. further process passes in a new condition of space *S* according to the density of distribution set by transitive function $f(x_{n+1} \mid x_n, a)$ (alternative assignation $f(\cdot|x_n, a))$. It is supposed that transitive function *f* and function of prize r depend only on a current condition *x* process and from the chosen alternative *a.* Any of these functions does not vary from a step to a step and does not depend on a process condition, on what or from the previous steps, from the alternatives chosen on these steps. Let's assume that in the beginning process is in set condition $x_0 \in S$. Choosing alternative $a_0 \in A,$, we will receive prize $r(x_0, a_0)$, and process passes in new condition X_1, it agree $f(\cdot x_0, a_0)$. Observing new condition $X_1 = x_1$ we choose alternative $a_1 \in A$, with prize $r(\cdot x_1, a_1)$. then process passes in new condition X_2 in conformity $f(|\cdot x_{1,} \ a_1)$. On $(n-1)$ - a step we will observe condition $X_{n-1} = x_{n-1}$, we choose alternative $a_{n-1} \in A$ and we receive prize $r(x_{n-1}, a_{n-1})$. Then process gets to final condition X_n agrees $f(.|x_{n-1}, a_{n-1})$. If to consider that in the end of process a price makes $r_0(x_n)$, that a problem consists in a choice of sequence of alternatives $a_0, a_1, ..., a_{n-1}$, defining the maximum average total prize which can be written down in a kind

$$V(x_0) = r(x_0, a_0) + E\langle [r(X_1, a_1) + ... + r(X_{n-1}, a_{n-1}) + r_0(X_n)] \tag{1}$$

By means of an induction method it is possible to define a kind of optimal procedure so back:

$$V_j(x) = \sup_{a \in A} \left\{ r(x, a) + \int_s V_{j-1}(x^i) f(x^i | x.a) \cdot d\mu(x^i) \right\}. \tag{2}$$

For any condition $x \in S$ put $V_0 = r_0(x)$' and will define functions $V_1, V_2, ..., V_n$ by the formula (2).

$$V_i(x) = \sup_{a \in A} \{ r(x, a) + E[r_0(X_n) | X_{n-1} = x, a_{n-1} = a] \} \tag{3}$$

There is the maximum average price received on last step of process, if $X_{n-1} = z$. For definition of the optimum initial Alternatives a_0 are necessary for beginning with the end sequence $a_{n,} a_{n-1,} \ ..., a_{0.}$ using the considered general statement of a problem for Markov process of the consecutive decision with final number of steps; we will define optimal control of crushing process as consecutive procedure also with final number of steps and square-law function of losses on each step.

Minimization of square-law function of losses leads to the optimal procedure of the decision based on linear functions from operated parameters, defined in an explicit form. Depending on number of operated parameters, one-dimensional and multidimensional procedures of search of optimum control differ.

Thus at the moment of a choice of value of management it is necessary and to provide enough possibility of supervision and an estimation of a condition of system which will be characterized by value large a crushing product. As shown above, value large scale a crushing product, defines the size of a target crack of the crushing unit, therefore, operating only the size of a target crack, it is possible to receive demanded value large scale a crushing product. From this point of view search of optimal procedure of management in the size of a target crack of the crushing unit will be the primary goal of optimal control provided that the system condition is unequivocally defined only by value size a product of crushing at any moment of management. Average demanded value size a product is the maximum average price received on last step of process, in the assumption that large a crushing product in an initial condition is a random variable with the normal aprioristic law of distribution. Estimating parameters normal a posterior law of distribution with a constant or a variable dispersion in all other conditions in the course of operated crushing, it is possible to define the optimum law of management of a target crack at faultless work of the automated control system as crushing, and also when at management the casual error which size depends on management size is brought. Let's consider statement of an one-dimensional problem of optimization of management by crushing manufacture and it is developed mathematical Optimal control models. Let $n-$ the set natural number and $X_1, \ldots, X_{n+1}$ - final sequence $n+1$ random variable - values large scale. Values of these sizes can be interpreted as a condition of the automated crushing manufacture, stochastic system being in this case on different steps consecutive $n-$ step-by-step managerial process (X_1 - initial condition of system, and $X_2 \ldots,$ X_{n+1} - system conditions on the subsequent steps). Thus the initial condition of system is defined initial size a product of crushing which, generally, is known and is a random variable with the normal law of distribution $X_1 = x_1$. In conformity, with properties Markov casual process, on some step *j* distribution of next condition X_{j+1} depends only on the present condition X_j and from value u_j of some material variable named management. At an assumption about absence of casual indignations of the system, the considered process can be described the following system of the equations:

$$X_{j+1} = \alpha_j X_j + \beta_j + u_j, \ (j = 1, \ \ldots, n), \tag{4}$$

Where α_j *and* β_j - constants, $\alpha_j \neq 0$, u_j - the value of management chosen after an estimation of condition X_j, The problem of optimal control of process of crushing of an initial material consists in definition of consecutive values $u_1, \ \ldots, \ u_n$ of management by optimal image. Thus the estimation of value of management is carried out so that the next condition of system bile X_{j+1} close to some demanded large value z_0. However in practical cases of an expense for management of crushing process can be that that there is necessary a compromise between cost of management and a damage, owing to a deviation of current value large a crushing product in a condition of system X_{j+1} from its demanded value z_0. In this connection, for optimization of value of management u_j on $j-m$ a step, we will estimate some function of damage $\lambda_j(x_j)$. From the previous remarks clearly that λ_o, $j-m$ step $(j = 1, \ldots n)$,) it is possible to present the general damage on in kind [4]:

$$\lambda_j(x_j) = q_j (X_{j+1} - z_0)^2 + r_j u_j^{\ 2}. \tag{5}$$

In expression (5) sums of squares first a member in the right part $q_j (X_{j+1} - z_0)^2$ defines A damage owing to a deviation of current value large scale, systems X_{j+1} $(j = 1, \ \ldots, \ \ n)$, corresponding to the next condition where $q_j \geq 0$ - non-negative weight function. The second member of the sum of expression of function of damage (5) $r_j u_j^2$ determines cost of a choice of value of management u_j on $j-m$ a step, where $r_j \geq 0$ - the non-negative constant w eight function setting the importance of cost of a choice of management. The general damage of all process is equal to the sum

$$L = \sum_{j=1}^{n} \lambda_j \tag{6}$$

Optimal sequence of values of management $u_1, \ \ldots, u_n$ is necessary to choose so that to minimize, average value of this sum. As in this problem of management there are only a final number of steps the optimal choice of sequence of values of management can be carried out by means of an induction method back. We will consider some step $j \, (j = 1, \ \ldots, \ n)$ Also we will admit that observed value $X_j = x_j$ and it was necessary to choose value u_j. the General damage from the remained steps of process it is equal

$$L_j(x_j) = \sum_{i=j}^{n} \lambda_j.$$

Thus, it is possible to present management of crushing process by some consecutive procedure with final number of steps and square-law function of losses on each step. Minimization of square-law function of losses leads as a result to reception of optimal procedure of management. Depending on number of operated parameters, one-dimensional and multidimensional procedures of search of optimum control differ. Thus at the moment of a choice of value of management it is necessary and to provide enough possibility of supervision and an estimation of a condition of system which will be characterized by value large a crushing product. As shown above, value size a crushing product defines the size of a target crack of the crushing unit; therefore, operating only the size of a target crack, it is possible to receive demanded value large a crushing product. From this point of view search of optimum procedure of management in the size of a target crack of the crushing unit will be the primary goal of optimum control provided that the system condition is unequivocally defined only by value large a product of crushing at any moment of management. Last step of managerial process (stop), is defined at the moment of equality, with certain accuracy, average current value large to demanded value. Estimating parameters normal posterior the law of distribution with a constant or a variable dispersion in all other conditions in the course of operated crushing, it is possible to define the optimum law of management of a target crack at faultless work of the automated control system as crushing, and also when at management the casual error which size depends on management size is brought.

3 Working out of Methods of One-Dimensional Optimal Control by Crushing Process

3.1 Optimal control without control system errors.

At one-dimensional optimal control large a crushing product we will believe according to defined above that the size of a target crack providing, as a result of the management, demanded value large a crushing product will be adjustable body. Let's consider discrete multistep-by-step process of the decision of this problem for the purpose of reception of optimal procedure of management for the cases, different an initial condition of system and process of a choice of the subsequent conditions up to realization of a rule of a stop of process. At a known initial condition of system we shall define $X_i = x_i$, value u_1 of management. In the conditions of uncertainty in relation to a kind of density of distribution large a crushing product as a continuous random variable, suppose, according to the equation (4) that the following condition X_2 has normal distribution with average $\alpha_1 x_1 + \beta_1 + u_1$ and dispersion σ_1^2. After supervision $X_2 = x_2$ we choose value u_2 of management. The next condition X_3 then is normally distributed with average $\alpha_2 x_2 + \beta_2 + u_2$ и dispersion $\sigma_2{}^2$. Considered process proceeds to final condition X_{n+1}.

Let's designate through $L_j(x_j)$ average value of the sum (6) if u_j and the further values of management get out in the optimal image. In particular $L_1(x_1)$ - an average minimum damage for all process, if initial condition X_i there is x_i. Let E_j at $j=1, \ldots, n$ marks a population mean calculated concerning distribution of random variable $X_{j+1,}$ at $X_j = x_j$ and a preset value of management u_j. If to define function $L_{n+1}(x_{n+1})$ as identical zero functions $L_{1,} \ \ldots, L_n$ should satisfy, for all $j=1, \ldots, n,$ to following expression

$$L_j(x_j) = \ln \underset{u_j}{f} E_j\left[\lambda_j + L_{j+1}(X_{j+1})\right] \tag{7}$$

Using a method of an induction and parity (6), expression (7) at $j=1, \ldots, n$ will present square-law function of a kind

$$L_j(x_j) = a_{j-1}(x_j - b_{j-1})^2 + c_{j-1}. \tag{8}$$

As function L_{n+1} identically is equal 0 it looks like (8) at

$$a_n = b_n = c_n = 0$$

Having chosen any constant $\delta,$ can write down for system of the equations (4) second the initial moment for size $(X_{j+1} - \delta)$ through dispersion so:

$$E_j\left[(X_{j+1} - \delta)^2\right] = (\alpha_j x_j + \beta_j + u_j - \delta)^2 + \sigma_j^2. \tag{9}$$

Hence, according to (5),

$$E_j(\lambda_j) = q_j(\alpha_j x_j + \beta_j + u_j - z_j)^2 + r_j u_j{}^2 + q_j\sigma_j^2. \tag{10}$$

Owing to equality (8),

$$E_j\left[L_{j+1}(X_{j+1})\right] = a_j(a_j x_j + \beta_j + u_1 - b_j)^2 + a_j\sigma_j{}^2 + c_j. \tag{11}$$

On sense of expression (7), optimal value u_j should deliver a minimum to the sum of dispersions (10) and (11).

$$L_j(x_j) = q_j(\alpha_j x_j + \beta_j + u_j - z_j)^2 + r_j u_j{}^2 + q_j\sigma_j^2 + \\ + a_j(\alpha_j x_j + \beta_j + u_j - b_j)^2. + \alpha_j\sigma_j{}^2 + c_j. \tag{12}$$

Let's define value of optimal control u_j, Take a private derivative on management from expression (12) and will equate to its zero: $\partial L_j(x_j)/\partial u_j = 0.$

Receive the equation:

$$u_j(q_j + r_j + \alpha_j) + (q_j + \alpha_j)(\alpha_j x_j + \beta_j) - q_j z_j - a_j b_j = 0$$

The decision of this equation gives optimal value of management u_j:

$$u_j = \frac{q_j z_j + a_j b_j - (q_j + \alpha_j)(\alpha_j x_j + \beta_j)}{q_j + \alpha_j + r_j} \tag{13}$$

Let's notice that the right part (13), at the accepted assumptions about character of development of process of optimal control, is linear function from a system condition during this moment $X_j = x_j$. Will substitute value (13) in expressions (8) and (10)._ After algebraic transformations will receive parities for definition of constants α_{j-1}, b_{j-1}, c_{j-1} the from (8) in a following kind:

$$\alpha_{j-1} = \frac{\alpha_j^2 r_j(q_j + \alpha_j)}{q_j + \alpha_j + r_j},$$

$$b_{j-1} = \frac{1}{\alpha_j}\left(\frac{q_j z_j + \alpha_j b_j}{q_j + \alpha_j} - \beta_j\right) \tag{14}$$

$$c_{j-1} = \frac{q_j\alpha_j(z_j - b_j)^2}{q_j + \alpha_j} + \left(q_j + \alpha_j\right)\sigma_j{}^2 + c_j$$

For definition under the formula (13) optimal importance's

$u_1, \ldots, u_n$, it is necessary for sequence of managements preliminary under formulas (14) to find all sequence of values $\alpha_{j-1}, b_{j-1}, c_{j-1} (j=1, \ldots, n)$, and moving back from $a_n = b_n = c_n = 0$. The minimum value of average damage $L_1(x_1)$ it is defined parity (8) at $j=1$. in expressions (13) and (14) dispersions $\sigma_j^2 (j=1, \ldots, n)$ do not appear. It means that at definition of optimal sequence of management's $u_j (j=1, \ldots, n)$ possible not to know values of dispersion large a crushing product, and to operate only with average value large, characterizing a system condition on each step. Therefore for this case optimum procedure of management can be realized, having constructed model of a population mean of optimum control large a crushing product. However actually the initial condition of operated crushing system X_0 represents casual value large.

The crushing product, having normal distribution with average x_0 and dispersion σ_0^2 which is known in advance (normal aprioristic distribution). In managerial process on each $j-$the step $(j=1, \ldots, n)$ condition of crushing system X_j also is a random variable large the crushing product, distributed normally with average x_j and dispersion $\sigma_j^2 = \sigma_0^2$ (normal a posteriori distribution) It means that the condition of crushing system in managerial process is observed with a constant error. According to system of the equations (4) for any choice of management u_j the following condition X_{j+1} is distributed under the normal law with average X_{j+1} and dispersion σ_{j+1}^2. For of admissible values $\alpha_j = 1$, $\beta_j = 0$ average value large and dispersions for $f+1$ a step will be are defined under formulas:

$$x_{i+1} = x_i + u_i \tag{15}$$

$$\sigma_{i+1}^2 = \sigma_i^2 \tag{16}$$

In an index point of process $j=1$ value calculation large x_i by sample casual values large from the normal aprioristic law of distribution large with known average value x_0 is carried out and dispersion σ_0^2. in the same point defines further optimal control u_i according to expressions (13 and 14). Values large x_j as random variable in all subsequent points of process are defined as average values of this random variable on the given step from normal posterior distributions with average value x_j (15) and dispersion σ_j^2. (16) through standard normal distribution with an average 0 and dispersion 1. At all $x(-\infty < x < +\infty)$ density standard normal distribution looks like:

$$\varphi(x) = (2\pi)^{\frac{1}{2}} \exp\left(-\frac{x^2}{2}\right). \tag{17}$$

For a considered case of a constancy of dispersion on all steps of management $(\sigma_j^2 = \sigma_0^2)$ sample of casual value large x_j, characterizing on j - ohm a management step, it is possible to carry out a system condition so

$$x_j = x_{j-1} + R\sigma_0, \tag{18}$$

Where σ_0 - average quadratic deviation large; R - random variable distributed under the standard normal law (17). Use of sample of casual value large $x_j (j=1, \ldots, n)$ on each step of managerial process with the specified parameters normal a posteriori Distributions allows, at definition of value of optimum control under the formula (13), to replace unknown exact value large x_j with average value large x_j^* random variable X_j on the given step.

3.2 Optimal control with control system errors.

Let's consider managerial process by crushing in which on each step of management the casual error which size depends on management size is brought in system. This error at definition of values of management u_j $(j=1, \ldots, n)$ affects not only on average value large a product of crushing for conditions X_{j+1} at a following step, but also on its dispersion. Let's describe managerial process in the presence of a casual error of management of the following system of the equations:

$$X_{j+1} = \alpha_j X_j + \beta_j + u_j + H_j u_j (j=1, \ldots, n). \tag{19}$$

In system of the equations (19) constant values α_j, β_j and management u_j affects the same sense, as in (4). Product of random variable H_j and managements u_{j^1} - answers an error brought by a control system. With population mean $E(H_j) = 0$ and dispersion $Var(H_j) = \eta_j^2$. Initial system it is fixed condition $X_1 = x_1$. At any set condition $X_1 = x_1$ and management u_j average value of next condition X_{j+1} is equal $\alpha_j X_j + \beta_j + u_j$, however the dispersion of management for this case is equal now $u_j^2 \eta_j^2 + \sigma_j^2$. Considered a problem is generalization of the problem solved in the previous point as it is possible to put $\eta_j^2 = 0$ at $j=1, \ldots, n$. If functions $L_j(x_j)$ at,

$j = 1, \ldots, n.$ are defined, how in the previous point they should satisfy to a parity (7). It is possible to show that $L_1(x_1)$-square-law function of a kind (8). The proof on an induction shows that optimal control u_j at $j = 1, \ldots, n.$ has a kind

$$u_j = \frac{q_j z_j + a_j b_j - (q_j + \alpha_j)(\alpha_j x_j + \beta_j)}{(q_j + \alpha_j)(1 + \eta_j^2) + r_j}$$

For considered problems $a_n = b_n = c_n = 0$ and at $j = 1, \ldots, n.$ factors a_j, b_j and c_j they are calculated so:

$$\alpha_{j-1} = \frac{\alpha_j^2 r_j (q_j + \alpha_j)\left[r_j + \eta_j^{\,2}(q_j + \alpha_j)\right]}{(q_j + \alpha_j)(1 + \eta_j^{\,2}) + r_j},$$

$$b_{j-1} = \frac{1}{\alpha_j}\left(\frac{q_j z_j + \alpha_j b_j}{q_j + \alpha_j} - \beta_j\right)$$

$$c_{j-1} = \frac{q_j \alpha_j (z_j - b_j)^2}{q_j + \alpha_j} + \left(q_j + \alpha_j\right)\sigma_j^{\,2} + c_j$$

If to assume that errors in definition large a product of crushing and a control system error are independent, value sample large, is carried out for $j = 1, \ldots, n.$ by the following formulas:

$$x_{j+1} = x_j + u_j\,, \quad x_j = x_{j-1} + R\sigma_j\,, \quad \sigma_j^{\,2} = \sigma_{j-1}^{\,2} + \eta_j^{\,2}$$

Where $\sigma_j^{\,,}$ - average quadratic deviation large; $R-$ random variable distributed under the standard normal law; $\eta_j^{\,2}$ - a dispersion of an error of a control system.

4 Conclusions

Thus, in work the problem of optimum control is qualified by process of crushing of initial materials, as a problem of consecutive decision-making which belongs to the class of statistical problems.

It is defined that key parameter of management of crushing units are the size of a target crack both cheek, and cone crushers or frequency of rotation of a splitting up cone and frequency swings a mobile cheek, and also is established that almost all parameters of working process including argue a crushing product, have stochastic character. Process of search of optimum control was approximated Markov by casual process. As prize function demanded value large crushing product was considered. Transitive function had been defined density of normal distribution large crushing product. The method of one-dimensional optimum control by process of crushing with the minimum square-law function of losses on each step of management is developed. At consecutive managerial process the stop was carried out at the moment of achievement, with the set accuracy, average value argue product of crushing of demanded value.

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